**Potential Energy**

- Potential energy is energy related to the configuration of a system in which the components of the system interact by forces
  - The forces are internal to the system
  - Can be associated with only specific types of forces acting between members of a system

**Gravitational Potential Energy**

- The system is the Earth and the book
- Do work on the book by lifting it slowly through a vertical displacement $\Delta r = \Delta y$
- The work done on the system must appear as an increase in the energy of the system

\[
W = (F_{app}) \cdot \Delta r
\]

\[
W = (mg) \cdot \left[ (y_f - y_i) \right]
\]

\[
W = mg y_f - mg y_i
\]

**Gravitational Potential Energy, final**

- The quantity $mg y$ is identified as the gravitational potential energy, $U_g$
  - $U_g = mg y$
- Units are joules (J)
- Is a scalar
- Work may change the gravitational potential energy of the system
  - $W_{net} = \Delta U_g$

**Click Question**

- John and Tom ski down from the same point of a mountain peak. John chose a slope of 10 degrees while Tom chose a slope of 5 degrees. They both reach the same ground level. What one has a higher potential energy in the end?
  - A. John
  - B. Tom
  - C. Same
  - D. Can not be determined
**Gravitational Potential Energy, Problem Solving**

- The gravitational potential energy depends only on the vertical height of the object above Earth’s surface.
- In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero.
  - The choice is arbitrary because you normally need the difference in potential energy, which is independent of the choice of reference configuration.

**Elastic Potential Energy**

- Elastic Potential Energy is associated with a spring.
- The force the spring exerts (on a block, for example) is $F_x = -kx$.
- The work done by an external applied force on a spring-block system is $W = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$.
  - The work is equal to the difference between the initial and final values of an expression related to the configuration of the system.

**Elastic Potential Energy, cont**

- This expression is the elastic potential energy: $U_e = \frac{1}{2}kx^2$.
- The elastic potential energy can be thought of as the energy stored in the deformed spring.
- The stored potential energy can be converted into kinetic energy.
- Observe the effects of different amounts of compression of the spring.

**Elastic Potential Energy, final**

- The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$).
  - The energy is stored in the spring only when the spring is stretched or compressed.
  - The elastic potential energy is a maximum when the spring has reached its maximum extension or compression.
  - The elastic potential energy is always positive.
  - $x^2$ will always be positive.

**Internal Energy**

- The energy associated with an object’s temperature is called its internal energy, $E_{int}$.
- In this example, the surface is the system.
- The friction does work and increases the internal energy of the surface.

**Conservative Forces**

- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
- The work done by a conservative force on a particle moving through any closed path is zero.
  - A closed path is one in which the beginning and ending points are the same.
Conservative Forces, cont

- Examples of conservative forces:
  - Gravity
  - Spring force
- We can associate a potential energy for a system with any conservative force acting between members of the system.
  - This can be done only for conservative forces.
  - In general: \( W_C = -\Delta U \)

Nonconservative Forces

- A nonconservative force does not satisfy the conditions of conservative forces.
- Nonconservative forces acting in a system cause a change in the mechanical energy of the system.

Nonconservative Forces, cont

- The work done against friction is greater along the brown path than along the blue path.
- Because the work done depends on the path, friction is a nonconservative force.

Conservative Forces and Potential Energy

- Define a potential energy function, \( U \), such that the work done by a conservative force equals the decrease in the potential energy of the system.
- The work done by such a force, \( F \), is
  \[
  W_C = \int F_x \, dx = -\Delta U
  \]
- \( \Delta U \) is negative when \( F \) and \( x \) are in the same direction.

Conservative Forces and Potential Energy – Check

- Look at the case of a deformed spring,
  \[
  F_x = -\frac{dU}{dx} = -\left( \frac{1}{2} k x^2 \right) = -kx
  \]
  - This is Hooke’s Law and confirms the equation for \( U \).
  - \( U \) is an important function because a conservative force can be derived from it.
**Energy Diagrams and Equilibrium**

- Motion in a system can be observed in terms of a graph of its position and energy.
- In a spring-mass system example, the block oscillates between the turning points, \(x = \pm x_{\text{max}}\).
- The block will always accelerate back toward \(x = 0\).

**Energy Diagrams and Stable Equilibrium**

- The \(x = 0\) position is one of **stable equilibrium**.
- Configurations of stable equilibrium correspond to those for which \(U(x)\) is a minimum.
- \(x = x_{\text{max}}\) and \(x = -x_{\text{max}}\) are called the turning points.

**Neutral Equilibrium**

- **Neutral equilibrium** occurs in a configuration when \(U\) is constant over some region.
- A small displacement from a position in this region will produce neither restoring nor disrupting forces.

**Energy Diagrams and Unstable Equilibrium**

- \(F = 0\) at \(x = 0\), so the particle is in equilibrium.
- For any other value of \(x\), the particle moves away from the equilibrium position.
- This is an example of **unstable equilibrium**.
- Configurations of unstable equilibrium correspond to those for which \(U(x)\) is a maximum.

**Potential Energy in Molecules**

- There is potential energy associated with the force between two neutral atoms in a molecule which can be modeled by the Lennard-Jones function:
  \[ U(x) = 4\varepsilon \left( \frac{\sigma}{x} \right)^{12} - \left( \frac{\sigma}{x} \right)^{6} \]

**Potential Energy Curve of a Molecule**

- Find the minimum of the function (take the derivative and set it equal to 0) to find the separation for stable equilibrium.
- The graph of the Lennard-Jones function shows the most likely separation between the atoms in the molecule (at minimum energy).
Types of Systems
- Nonisolated systems
  - Energy can cross the system boundary in a variety of ways
  - Total energy of the system changes
- Isolated systems
  - Energy does not cross the boundary of the system
  - Total energy of the system is constant

Ways to Transfer Energy Into or Out of A System
- **Work** – transfers by applying a force and causing a displacement of the point of application of the force
- **Mechanical Waves** – allow a disturbance to propagate through a medium
- **Heat** – is driven by a temperature difference between two regions in space

More Ways to Transfer Energy Into or Out of A System
- **Matter Transfer** – matter physically crosses the boundary of the system, carrying energy with it
- **Electrical Transmission** – transfer is by electric current
- **Electromagnetic Radiation** – energy is transferred by electromagnetic waves

Examples of Ways to Transfer Energy
- a) Work
- b) Mechanical Waves
- c) Heat

Examples of Ways to Transfer Energy, cont.
- d) Matter transfer
- e) Electrical Transmission
- f) Electromagnetic radiation

Conservation of Energy
- **Energy is conserved**
  - This means that energy cannot be created nor destroyed
  - If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer
**Conservation of Energy, cont.**

- Mathematically, $\Delta E_{\text{system}} = \sum T$
  - $E_{\text{system}}$ is the total energy of the system
  - $T$ is the energy transferred across the system boundary
  - Established symbols: $T_{\text{work}} = W$ and $T_{\text{heat}} = Q$
  - Others just use subscripts
- The Work-Kinetic Energy theorem is a special case of Conservation of Energy
  - The full expansion of the above equation gives $\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{\text{MW}} + T_{\text{MT}} + T_{\text{ET}} + T_{\text{ER}}$

**Isolated System**

- For an isolated system, $\Delta E_{\text{mech}} = 0$
  - Remember $E_{\text{mech}} = K + U$
  - This is **conservation of energy** for an isolated system with no nonconservative forces acting
- If nonconservative forces are acting, some energy is transformed into internal energy
- Conservation of Energy becomes $\Delta E_{\text{system}} = 0$
  - $E_{\text{system}}$ is all kinetic, potential, and internal energies
  - This is the most general statement of the isolated system model

**Isolated System, cont**

- The changes in energy can be written out and rearranged
  - $K_f + U_f = K_i + U_i$
  - Remember, this applies only to a system in which conservative forces act

**Example – Free Fall**

- Determine the speed of the ball at $y$ above the ground
- Conceptualize
  - Use energy instead of motion
- Categorize
  - System is isolated
  - Only force is gravitational which is conservative

**Example – Free Fall, cont**

- Analyze
  - Apply Conservation of Energy
  - $K_f + U_f = K_i + U_i$
  - $K_i = 0$, the ball is dropped
  - Solving for $v_f$
    $$v_f = \sqrt{v_i^2 + 2g(h-y)}$$
- Finalize
  - The equation for $v_f$ is consistent with the results obtained from kinematics

**Example – Spring Loaded Gun**

- Conceptualize
  - The projectile starts from rest
  - Speeds up as the spring pushes against it
  - As it leaves the gun, gravity slows it down
- Categorize
  - System is projectile, gun, and Earth
  - Model as a system with no nonconservative forces acting
Example – Spring Gun, cont
- Analyze
  - Projectile starts from rest, so $K_i = 0$
  - Choose zero for gravitational potential energy where projectile leaves the gun
  - Elastic potential energy will also be 0 here
  - After the gun is fired, the projectile rises to a maximum height, where its kinetic energy is 0
- Finalize
  - Did the answer make sense
  - Note the inclusion of two types of potential energy

Kinetic Friction
- Kinetic friction can be modeled as the interaction between identical teeth
- The frictional force is spread out over the entire contact surface
- The displacement of the point of application of the frictional force is not calculable

Work – Kinetic Energy
- Is valid for a particle or an object that can be modeled as an object
- When a friction force acts, you cannot calculate the work done by friction
- However, Newton’s Second Law is still valid even though the work-kinetic energy theorem is not valid

Work – Kinetic Energy With Friction
- In general, if friction is acting in a system:
  - $\Delta K = \sum W_{\text{other forces}} - f_k d$
  - This is a modified form of the work – kinetic energy theorem
  - Use this form when friction acts on an object
  - If friction is zero, this equation becomes the same as Conservation of Mechanical Energy

Including Friction, final
- A friction force transforms kinetic energy in a system to internal energy
- The increase in internal energy of the system is equal to its decrease in kinetic energy
- $\Delta E_{\text{int}} = f_k d$

Example – Block on Rough Surface
- The block is pulled by a constant force over a rough horizontal surface
- Conceptualize
  - The rough surface applies a friction force on the block
  - The friction force is in the direction opposite to the applied force
Example – Block-spring System

- The problem
  - The mass is attached to a spring, the spring is compressed and then the mass is released
  - A constant friction force acts
- Conceptualize
  - The block will be pushed by the spring and move off with some speed
- Categorize
  - Block and surface is the system
  - System is nonisolated

Clicker Problem

- For the system shown on right, after the block is released from x=-a, the maximum kinetic energy occurs at
  A. x=-a
  B. x=0
  C. x=a/2
  D. kinetic energy is a constant
  E. x=a

Example – Spring-block, cont

- Analyze
  - Evaluate $f_k \, d$
  - Evaluate $\delta W_{\text{other forces}}$
- Finalize
  - Think about the result

Adding Changes in Potential Energy

- If friction acts within an isolated system
  $\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k \, d$
  - $\Delta U$ is the change in all forms of potential energy
- If friction acts within a nonisolated system
  $\Delta E_{\text{mech}} = -f_k \, d + \Sigma W_{\text{other forces}}$

Example – Ramp with Friction

- Problem: the crate slides down the rough ramp
  - Find speed at bottom
- Conceptualize
  - Energy considerations
- Categorize
  - Identify the crate, the surface, and the Earth as the system
  - Isolated system with nonconservative force acting

Example – Ramp, cont

- Analyze
  - Let the bottom of the ramp be y = 0
  - At the top: $E_i = K_i + U_{gi} = 0 + mgy_i$
  - At the bottom: $E_f = K_f + U_{gf} = \frac{1}{2} m v_f^2 + mgy_f$
  - Then $\Delta E_{\text{mech}} = E_f - E_i = -f_k \, d$
  - Solve for $v_f$
- Finalize
  - Could compare with result if ramp was frictionless
  - The internal energy of the system increased
Example – Spring Mass Collision

- Without friction, the energy continues to be transformed between kinetic and elastic potential energies and the total energy remains the same.
- If friction is present, the energy decreases.
  \[ \Delta E_{\text{mech}} = -f_k d \]

Example – Spring Mass, 2

- Conceptualize
  - All motion takes place on a horizontal plane.
  - So no changes in gravitational potential energy.

- Categorize
  - The system is the block and the system.
  - Without friction, it is an isolated system with no nonconservative forces.

- Analyze
  - Before the collision, the total energy is kinetic.

Example – Connected Blocks

- Conceptualize
  - Configurations of the system when at rest are good candidates for initial and final points.

- Categorize
  - The system consists of the two blocks, the spring, and Earth.
  - System is isolated with a nonconservative force acting.

Example – Blocks, cont

- Categorize, cont
  - Gravitational and potential energies are involved.
  - The kinetic energy is zero if our initial and final configurations are at rest.
  - Model the sliding block as a particle in equilibrium in the vertical direction.

- Analyze
  - Two forms of potential energy are involved.

Problem – Spring Mass 3

- Analyze
  - Before the collision, the total energy is kinetic.
  - When the spring is totally compressed, the kinetic energy is zero and all the energy is elastic potential.
  - Total mechanical energy is conserved.

- Finalize
  - Decide which root has physical meaning.

Problem – Spring Mass 4

- Now add friction
  - Categorize
    - Now is isolated with nonconservative force.
  - Analyze
    - Use \( \Delta E_{\text{mech}} = -f_k d \).
  - Finalize
    - The value is less than the case for no friction.
      - As expected.
**Connected Blocks, cont**
- Analyze, cont
  - Block 2 undergoes a change in gravitational potential energy
  - The spring undergoes a change in elastic potential energy
  - The coefficient of kinetic energy can be measured
- Finalize
  - This allows a method for measuring the coefficient of kinetic energy

**Instantaneous Power**
- Power is the time rate of energy transfer
- The *instantaneous power* is defined as
  \[ \bar{\nu} = \frac{dE}{dt} \]
- Using work as the energy transfer method, this can also be written as
  \[ \bar{\nu}_{avg} = \frac{W}{\Delta t} \]

**Power**
- The time rate of energy transfer is called power
- The average power is given by
  \[ \bar{P} = \frac{W}{\Delta t} \]
  - when the method of energy transfer is work

**Instantaneous Power and Average Power**
- The instantaneous power is the limiting value of the average power as \( \Delta t \) approaches zero
  \[ \bar{\nu} = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = F \cdot \frac{dv}{dt} = F \cdot \dot{v} \]
- The power is valid for any means of energy transfer

**Units of Power**
- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg \cdot m^2 / s^2
- A unit of power in the US Customary system is horsepower
  - 1 hp = 746 W
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 \times 10^6 J